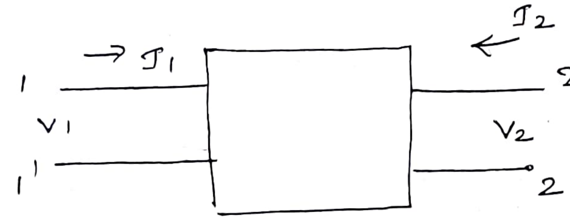
Aswathy madam

$$\alpha_{21}(s) = \frac{I_2(s)}{I_1(s)} = 0$$

MODULE- V

Two Port Networks.

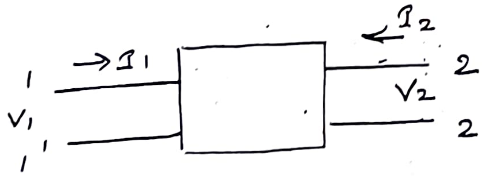


The terminals 1-1' together constitute a port. Similarly the terminals 2-2' constitute another port. Two ports containing no sources in their branches are called passive ports, among them are power transmission lines and transformers.

Two ports containing sources in their branches are called active ports.

A voltage and current is attached to each of these 2 ports. The voltage & current at the input terminals are V_1 & I_1 whereas V_2 & I_2 are specified at the output port. It is also assumed that currents I_1 & I_2 are entering into the network at the upper terminals 1 & 2 respectively. The variables of the two. port network are V_1, V_2 & I_1, I_2. Two of these are dependent variables the other two are independent variables. The number of possible combinations generated by the 4 variables, taken 2 at a time is 6. Thus there are 6 possible sets of equations describing a two-port ~~pee~~ network.

(1) Open Circuit Impedance (Z) parameters



A general linear two port network which does not contain any independent sources is shown in fig. The Z parameters of a two port for the positive directions of voltages & currents may be defined by expressing the port voltages V_1 & V_2 in terms of the currents I_1 & I_2. Here V_1 & V_2 are dependent variables & I_1 & I_2 are independent variables. Thus the voltage at port 1-1' is the response produced by the two currents I_1 & I_2

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

where $Z_{11}, Z_{12}, Z_{21}, Z_{22}$ are ~~called~~ the network functions & are called impedance parameters. These parameters can be represented by matrices

$$[V] = [Z][I]$$

where $[V]$ is the column matrix $= \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$

$[Z]$ is the square matrix $= \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$

$[I]$ is the ~~row~~ column matrix $= \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

Pavithra

The individual Z parameters for a given network can be defined by setting each of the port currents equal to zero. Suppose port 2-2' is left open-circuited, then $I_2 = 0$

Then $$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$

where Z_{11} is the driving point impedance at port 1-1' with port 2-2' open circuited. It is called the open circuit input impedance.

Similarly $$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

where Z_{21} is the transfer impedance at port 1-1' ~~with~~ with port 2-2' open circuited. It is also called the open circuit forward transfer impedance.

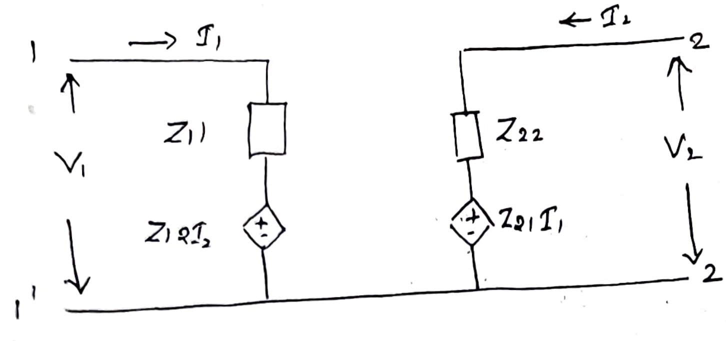
$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

where Z_{12} is the transfer impedance at port 2-2' with port 1-1' open circuited. It is also called the open circuit reverse transfer impedance.

$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

where Z_{22} is the open circuit driving point impedance at port 2-2' with port 1-1' open circuited. It is also called open circuit output impedance.

Equivalent Circuit

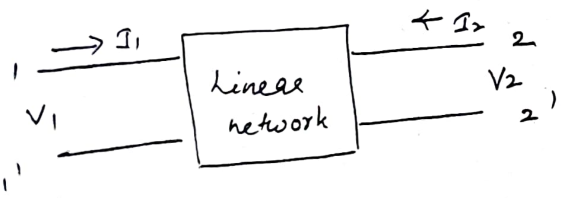


* If the network under study is reciprocal or bilateral, then in accordance with the reciprocity principle

$$\left.\frac{V_2}{I_1}\right|_{I_2=0} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

or $Z_{21} = Z_{12}$

Short Circuit Admittance (Y) Parameters



The Y parameters of a two port for the positive directions of voltages & currents may be defined by expressing the port currents I_1 & I_2 in terms of the voltages V_1 & V_2. Here I_1, I_2 are dependent variables & V_1 & V_2 are independent variables. I_1 may be considered to be superposition of two components. One caused by V_1 & the other by V_2.

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

Y_{11}, Y_{12}, Y_{21} & Y_{22} are the network function & are also called the admittance (Y) parameters. These parameters can be represented by matrices as follows

$$[I] = [Y][V]$$

where $I = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$

$$Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

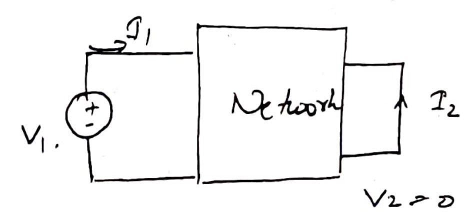
$$V = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Thus $$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

If either V_1 or V_2 is zero, the four parameters may be defined in terms of a voltage & current

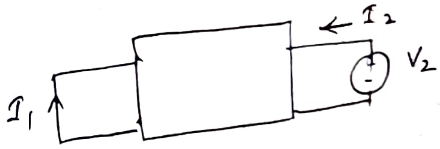
thus

$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2 = 0}$$



$$Y_{21} = \frac{I_2}{V_1}\Big|_{V_2 = 0}$$

$$Y_{12} = \frac{I_1}{V_2}\bigg|_{V_1=0}$$



$$Y_{22} = \frac{I_2}{V_2}\bigg|_{V_1=0}$$

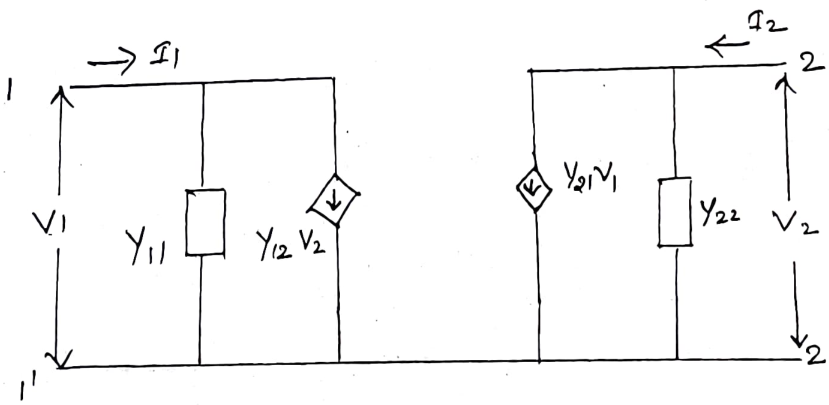
Y_{11} = short circuit input admittance.

Y_{21} = short circuit forward transfer admittance.

Y_{12} = short circuit reverse transfer admittance

Y_{22} = short circuit driving point admittance.

Equivalent circuit



If the network under study is reciprocal or bilateral

then $\frac{I_1}{V_2}\bigg|_{V_1=0} = \frac{I_2}{V_1}\bigg|_{V_2=0}$

or $Y_{12} = Y_{21}$.

All parameters have the dimensions of admittance which are obtained by short circuiting either the output or the input.

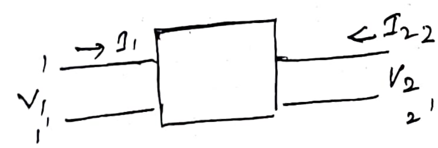
(3) Transmission (ABCD) Parameters.

Transmission parameters are widely used in transmission line theory & cascade networks. The transmission parameters serve to relate the voltage & current at one port to voltage & current at the other port. Their first use was in the analysis of power transmission lines where they are also known as general circuit parameters

$$V_1 = AV_2 - BI_2 \quad \text{---}(1)$$

$$I_1 = CV_2 - DI_2 \quad \text{---}(2)$$

The negative sign for the second term in the above equation arises from 2 different conventions in assigning a positive direction to I_2. In power transmission problems it is conventional for the current to be assigned a reference direction which is opposite to that given in fig.



Thus the negative signs in eqn (1) & (2) are for I_2 and not for B & D. The input variables V_1 & I_1 at port 1-1' usually called sending end are expressed in terms of the output variable V_2 & I_2 at port 2-2' called the receiving end. These parameters provide a direct relationship between input & output.

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

The matrix $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is called the transmission matri:

→ With port 2-2' open, i.e. $I_2 = 0$, applying a voltage V_1 at the port 1-1'

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \quad \& \quad C = \left.\frac{I_1}{V_2}\right|_{I_2=0}.$$

$$\frac{1}{A} = \left.\frac{V_2}{V_1}\right|_{I_2=0} = G_{21} \qquad \frac{1}{C} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = Z_{21}$$

where $\frac{1}{A}$ is called the open circuit voltage gain

$\frac{1}{C}$ is an open circuit transfer impedance

→ With port 2-2' short-circuited i.e. $V_2 = 0$, applying voltage V_1 at port 1-1'

$$-B = \left.\frac{V_1}{I_2}\right|_{V_2=0} \quad \& \quad -D = \left.\frac{I_1}{I_2}\right|_{V_2=0}.$$

$$-\frac{1}{B} = \left.\frac{I_2}{V_1}\right|_{V_2=0} = Y_{21} \quad \text{short circuit transfer admittance}$$

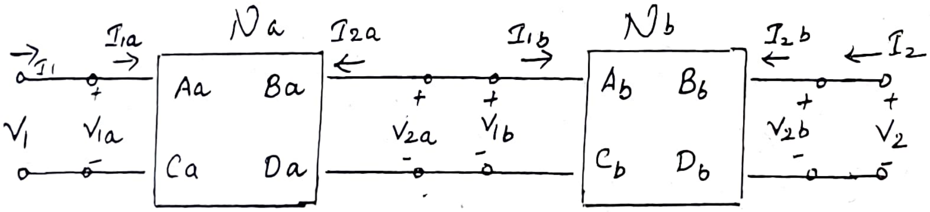
$$-\frac{1}{D} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \alpha_{21} \quad \text{short circuit current gain (dimensionless)}$$

Cascade connection.

The main use of the transmission matrix is in dealing with a cascade connection of two-port network. Consider N_a & N_b connected in cascade with port voltages & currents as indicated in fig.

$$\begin{bmatrix} V_{1a} \\ I_{1a} \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} V_{2a} \\ -I_{2a} \end{bmatrix}$$

$$\begin{bmatrix} V_{1b} \\ I_{1b} \end{bmatrix} = \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$



$V_1 = V_{1a}$ $\quad I_{2a} = -I_{1b}$ $\quad I_{2b} = I_2$

$I_1 = I_{1a}$ $\quad V_{2a} = V_{1b}$ $\quad V_{2b} = V_2$.

$$\therefore \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_{2b} \\ -I_{2b} \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}.$$

where $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is the transmission parameter matrix for the overall network.

Thus the transmission matrix of a cascade two port network is the product of transmission matrices of the individual two-port networks. This property is used in the design of telephone systems, microwave networks radars, etc.

$$A = \frac{Z_{11}}{Z_{21}}, \quad B = \frac{\Delta z}{Z_{21}}, \quad C = \frac{1}{Z_{21}}, \quad D = \frac{Z_{22}}{Z_{21}}$$

Inverse Transmission Parameters $(A' B' C' D')$

The voltage & current at the receiving end can also be expressed in terms of the sending end voltage & current. If the voltage & current at port 2-2' is expressed in terms of voltage & current at 1-1', then

$$V_2 = A'V_1 - B'I_1$$

$$I_2 = C'V_1 - D'I_1$$

The coefficients A', B', C', D' in the above equations are called inverse transmission parameters.

Also $A' = \left.\frac{V_2}{V_1}\right|_{I_1=0}$, $C' = \left.\frac{I_2}{V_1}\right|_{I_1=0}$.

$B' = \left.\frac{-V_2}{I_1}\right|_{V_1=0}$, $D' = \left.\frac{-I_2}{I_1}\right|_{V_1=0}$

So

$$AD - BC = \frac{Z_{12}}{Z_{21}}$$

If $Z_{12} = Z_{21}$, as is the case for a reciprocal network

then $AD - BC = 1$.

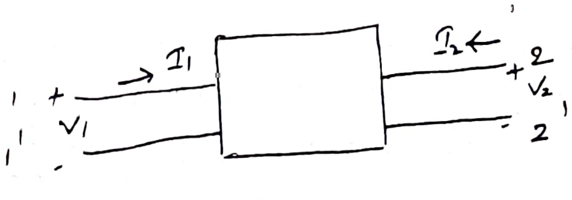
and for inverse transmission parameters

$A'D' - B'C' = 1$.

4) The Hybrid Parameters. (h)

The hybrid parameters find usage in electronic circuits, especially for constructing models for transistors. as these parameters can be most conveniently measured. The hybrid matrices describe a two-port, when the voltage of one port and the current of other port are taken as the independent variables. If the voltage at port 1-1' & current at port 2-2' are taken as dependent variables then

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2 .$$



The coefficients in the above equations are called hybrid parameters.

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

→ When $V_2 = 0$, the port 2-2' is short-circuited.

$h_{11} = \left[\frac{V_1}{I_1}\right]_{V_2=0} = \frac{1}{Y_{11}}$ → short circuit input impedance.

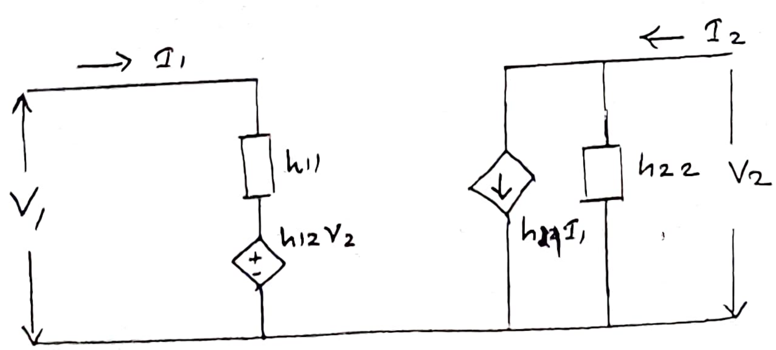
$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} = \frac{Y_{21}}{Y_{11}}$ → short circuit forward current gain

→ When port 1-1' is open circuited $I_1 = 0$.

$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} = \frac{Z_{12}}{Z_{22}}$ → open circuit reverse voltage gain

$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} = \frac{1}{Z_{22}}$ → open circuit ~~op~~ out-put admittance

Since h parameters represent dimensionally an impedance an admittance, a voltage gain and a current gain, these are called hybrid parameters.



Inverse hybrid parameters (g)

In this the current at the input port I_1 and the voltage at the output port V_2 can be expressed in terms of I_2 & V_1.

$$I_1 = g_{11} V_1 + g_{12} I_2$$
$$V_2 = g_{21} V_1 + g_{22} I_2 .$$

The coefficients in the above equations are called the inverse hybrid parameters

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} .$$

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}^{-1} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}$$

The individual g parameters may be defined by letting $I_2 = 0$ & $V_1 = 0$.

Thus when $I_2 = 0$

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0} = \frac{1}{Z_{11}}$$ → open circuit input admittance.

$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0}$$ = Open circuit voltage gain

when $V_1 = 0$

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0}$$, short circuit reverse current gain.

$$g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$$ = short circuit output impedance $\frac{1}{Y_{22}}$

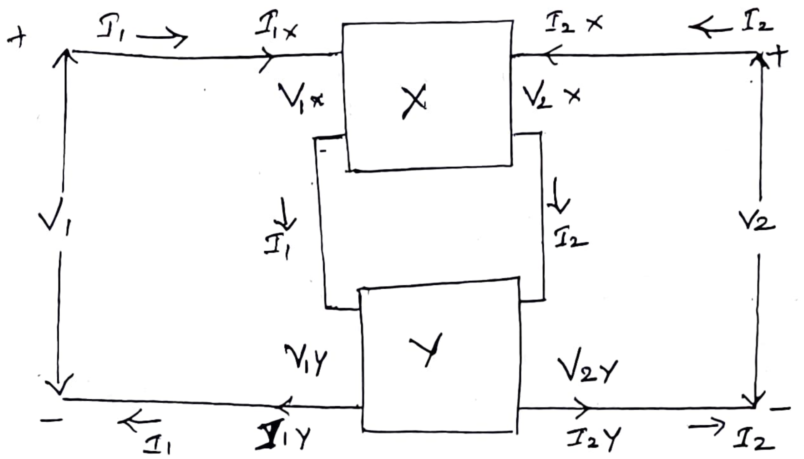
Name	Function		Equation
	Express	In terms of	
Open-circuit impedance Z	V_1, V_2	I_1, I_2	$V_1 = Z_{11} I_1 + Z_{12} I_2$ $V_2 = Z_{21} I_1 + Z_{22} I_2$
Short-circuit admittance Y	I_1, I_2	V_1, V_2	$I_1 = Y_{11} V_1 + Y_{12} V_2$ $I_2 = Y_{21} V_1 + Y_{22} V_2$
Transmission ABCD	V_1, I_1	V_2, I_2	$V_1 = AV_2 - BI_2$ $I_1 = CV_2 - DI_2$
Inverse transmission $A'B'C'D'$	V_2, I_2	V_1, I_1	$V_2 = A'V_1 - B'I_1$ $I_2 = C'V_1 - D'I_1$
Hybrid	V_1 I_2	I_1, V_2	$V_1 = h_{11} I_1 + h_{12} V_2$ $I_2 = h_{21} I_1 + h_{22} V_2$
Inverse hybrid	I_1, V_2	V_1, I_2	$I_1 = g_{11} V_1 + g_{12} I_2$ $V_2 = g_{21} V_1 + g_{22} I_2$

Analysis of Interconnected Two-port Networks.

Series connection of a Two-port Network.

Z parameter can be ~~on~~ used to describe the parameters of series connected two-port networks & Y parameters can be used to describe parameters of parallel connected two port networks.

A series connected two port network is shown in fig.



Consider two, two port networks, connected in series as shown. If each port has a common reference node for its input and output and if these references are connected together then the equations of the networks X & Y in terms of Z parameter are:

$$V_{1X} = Z_{11X} I_{1X} + Z_{12X} I_{2X} \quad \text{— } ①$$

$$V_{2X} = Z_{21X} I_{1X} + Z_{22X} I_{2X} \quad \text{— } ②$$

$$V_{1Y} = Z_{11Y} I_{1Y} + Z_{12Y} I_{2Y} \quad \text{— } ③$$

$$V_{2Y} = Z_{21Y} I_{1Y} + Z_{22Y} I_{2Y} \quad \text{— } ④$$

From the inter connection of the network it is clear that

$$I_1 = I_{1X} = I_{1Y} \; ; \quad I_2 = I_{2X} = I_{2Y}$$

$$V_1 = V_{1X} + V_{1Y} \text{ — } ⑤ \; ; \quad V_2 = V_{2X} + V_{2Y} \text{ — } ⑥$$

Adding ① & ③

$$\therefore V_1 = Z_{11X} I_{1X} + Z_{12X} I_{2X} + Z_{11Y} I_{1Y} + Z_{12Y} I_{2Y}$$

$$= Z_{11X} I_1 + Z_{12X} I_2 + Z_{11Y} I_1 + Z_{12Y} I_2$$

$$= (Z_{11X} + Z_{11Y}) I_1 + (Z_{12X} + Z_{12Y}) I_2$$

Adding ② & ④ , ⑥ becomes.

$$V_2 = Z_{21x} I_{1x} + Z_{22x} I_{2x} + Z_{21y} I_{1y} + Z_{22y} I_{2y}$$

$$= Z_{21x} I_1 + Z_{22x} I_2 + Z_{21y} I_1 + Z_{22y} I_2$$

$$V_2 = (Z_{21x} + Z_{21y}) I_1 + (Z_{22x} + Z_{22y}) I_2$$

$\therefore$

$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

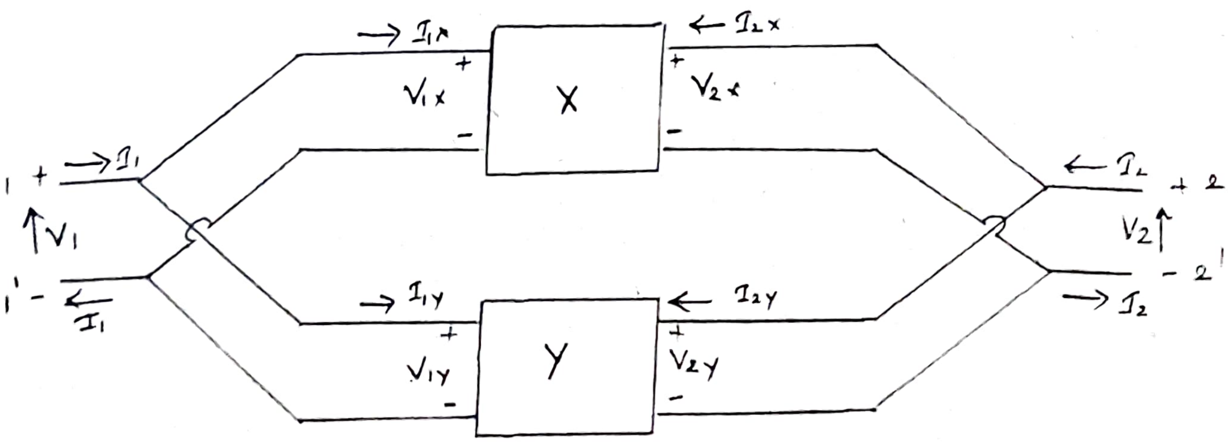
$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

where $Z_{11} = Z_{11x} + Z_{11y}$; $Z_{12} = Z_{12x} + Z_{12y}$

$Z_{21} = Z_{21x} + Z_{21y}$; $Z_{22} = Z_{22x} + Z_{22y}$.

Each Z parameter of the series network is given as the sum of the corresponding parameters of the individual networks.

Parallel Connection of Two Two-port Networks.



Consider two two-port networks connected in parallel as shown in fig. If each two-port has a reference node that is common to its input & output port, and if the two ports are connected so that they have a common reference node, then the equatic of the networks X & Y in terms of Y parameters are given by.

$$I_{1X} = Y_{11X} V_{1X} + Y_{12X} V_{2X} \quad \text{—} ①$$

$$I_{2X} = Y_{21X} V_{1X} + Y_{22X} V_{2X} \quad \text{—} ②$$

$$I_{1Y} = Y_{11Y} V_{1Y} + Y_{12Y} V_{2Y} \quad \text{—} ③$$

$$I_{2Y} = Y_{21Y} V_{1Y} + Y_{22Y} V_{2Y} \quad \text{—} ④$$

From the interconnection of the networks, it is clear that

$$V_1 = V_{1X} = V_{1Y} \; ; \quad V_2 = V_{2X} = V_{2Y}$$

$$I_1 = I_{1X} + I_{1Y} \text{ —} ⑤ \; ; \quad I_2 = I_{2X} + I_{2Y} \text{ —} ⑥$$

∴ ~~Ans~~

Adding ① & ③ and substituting in ⑤

$$I_1 = Y_{11X} V_{1X} + Y_{12X} V_{2X} + Y_{11Y} V_{1Y} + Y_{12Y} V_{2Y}$$

$$= (Y_{11X} + Y_{11Y}) V_1 + (Y_{12X} + Y_{12Y}) V_2$$

$$I_2 = Y_{21X} V_{1X} + Y_{22X} V_{2X} + Y_{21Y} V_{1Y} + Y_{22Y} V_{2Y}$$

$$= (Y_{21X} + Y_{21Y}) V_1 + (Y_{22X} + Y_{22Y}) V_2$$

The describing equations for the parallel connected two port networks are

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$\therefore$ $Y_{11} = Y_{11x} + Y_{11y}$.

$Y_{12} = Y_{12x} + Y_{12y}$.

$Y_{21} = Y_{21x} + Y_{21y}$

$Y_{22} = Y_{22x} + Y_{22y}$.

Conversion formulas

$\underline{\underline{Z}}$

(1) Z in terms of Y.

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1}$$

$$[A]^{-1} = \frac{\operatorname{adj} A}{|A|} \quad \rightarrow \operatorname{adj} A = \text{cofactor } A^{T}$$

$$\operatorname{adj}(Y) = \begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}$$

$$|A| = Y_{11}Y_{22} - Y_{12}Y_{21} = \Delta Y$$

$$\therefore [Y]^{-1} = \frac{\begin{pmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{pmatrix}}{\Delta Y}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{Y_{22}}{\Delta Y} & \frac{-Y_{12}}{\Delta Y} \\ \frac{-Y_{21}}{\Delta Y} & \frac{Y_{11}}{\Delta Y} \end{bmatrix}$$

$$\therefore Z_{11} = \frac{Y_{22}}{\Delta Y}$$

$$Z_{12} = \frac{-Y_{12}}{\Delta Y}$$

$$Z_{21} = \frac{-Y_{21}}{\Delta Y}$$

$$Z_{22} = \frac{Y_{11}}{\Delta Y}.$$

2) Z in terms of ABCD Parameters:

$$V_1 = AV_2 - BI_2 \quad -(1)$$

$$I_1 = CV_2 - DI_2 \quad -(2)$$

From (2)

$$CV_2 = I_1 + DI_2$$

$$V_2 = \frac{1}{C} I_1 + \frac{D}{C} I_2. \quad -(3)$$

Sub (3) in (1)

Then e

$$V_1 = A\left(\frac{1}{C} I_1 + \frac{D}{C} I_2\right) - BI_2$$

$$= \frac{A}{C} I_1 + \frac{AD}{C} I_2 - BI_2$$

$$V_1 = \frac{A}{C} I_1 + \frac{(AD - BC)}{C} I_2 \quad -(4)$$

From (4)

$$Z_{11} = \frac{A}{C} \qquad Z_{12} = \frac{AD-BC}{C}$$

From (3)

$Z_{21} = \frac{1}{C}$, $Z_{22} = \frac{D}{C}$.

(3) Z parameters in terms of Hybrid Parameters

$V_1 = h_{11} I_1 + h_{12} V_2$ —(1)

$I_2 = h_{21} I_1 + h_{22} V_2$ —(2).

Z parameters

$V_1 = Z_{11} I_1 + Z_{12} I_2$ —(a)

$V_2 = Z_{21} I_1 + Z_{22} I_2$. —(b)

From (2)

$V_2 = \frac{1}{h_{22}} I_2 - \frac{h_{21}}{h_{22}} I_1$ —(3)

Sub (3) in (1).

$V_1 = h_{11} I_1 + h_{12} \left(\frac{1}{h_{22}} I_2 - \frac{h_{21}}{h_{22}} I_1 \right)$

$= h_{11} I_1 + \frac{h_{12}}{h_{22}} I_2 - \frac{h_{12} h_{21}}{h_{22}} I_1$

$V_1 = \frac{h_{11} h_{22} - h_{12} h_{21} I_1}{h_{22}} + \frac{h_{12}}{h_{22}} I_2$.

$V_1 = \frac{\Delta h}{h_{22}} I_1 + \frac{h_{12}}{h_{22}} I_2$ —(4)

Comparing (a) & (4)

$Z_{11} = \frac{\Delta h}{h_{22}}$, $Z_{12} = \frac{h_{12}}{h_{22}}$.

Comparing (b) & (3)

$Z_{21} = \frac{-h_{21}}{h_{22}}$, $Z_{22} = \frac{1}{h_{22}}$

ABCD

(i) ABCD in terms of z parameters

(i) ABCD $\Rightarrow$ Z

$V_1 = AV_2 - BI_2$ — (a)

$I_1 = CV_2 - DI_2$ — (b)

$V_1 = Z_{11} I_1 + Z_{12} I_2$ — (1)

$V_2 = Z_{21} I_1 + Z_{22} I_2$ — (2)

From (2)

$Z_{21} I_1 = V_2 - Z_{22} I_2$

$I_1 = \frac{1}{Z_{21}} V_2 - \frac{Z_{22}}{Z_{21}} I_2$ — (3)

Sub (3) in (1)

$V_1 = Z_{11} \left(\frac{1}{Z_{21}} V_2 - \frac{Z_{22}}{Z_{21}} I_2 \right) + Z_{12} I_2$

$= \frac{Z_{11}}{Z_{21}} V_2 - \frac{Z_{11} Z_{22} I_2}{Z_{21}} + Z_{12} I_2$

$V_1 = \frac{Z_{11}}{Z_{21}} V_2 - \left(\frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{21}} \right) I_2$ — (4)

Comparing

(a) & (4)

$A = \frac{Z_{11}}{Z_{21}}$, $B = \frac{Z_{11} Z_{22} - Z_{12} Z_{21}}{Z_{21}}$

$B = \frac{\Delta Z}{Z_{21}}$

Comparing ⑥ & ③

$C = \frac{1}{Z_{21}}$, $D = \frac{Z_{22}}{Z_{21}}$

(2) ABCD in terms of Y parameters

$V_1 = AV_2 - BI_2$ — ①

$I_1 = CV_2 - DI_2$ — ②

$I_1 = Y_{11} V_1 + Y_{12} V_2$ — ③

$I_2 = Y_{21} V_1 + Y_{22} V_2$ — ④

from ④

$$V_1 = \frac{1}{Y_{21}} I_2 - \frac{Y_{22}}{Y_{21}} V_2 \quad \text{— ⑤}$$

Sub ⑤ in ③

$$I_1 = Y_{11}\left(\frac{1}{Y_{21}} I_2 - \frac{Y_{22}}{Y_{21}} V_2\right) + Y_{12} V_2$$

$$= \frac{Y_{11} I_2}{Y_{21}} - \frac{Y_{11} Y_{22}}{Y_{21}} V_2 + Y_{12} V_2$$

$$I_1 = \frac{Y_{11}}{Y_{21}} I_2 - \frac{(Y_{11}Y_{22} - Y_{21}Y_{12})}{Y_{21}} V_2$$

$$= -\frac{(Y_{11}Y_{22} - Y_{21}Y_{12})}{Y_{21}} V_2 + \frac{Y_{11}}{Y_{21}} I_2 \quad \text{— ⑥}$$

Comparing ⑤ & ①

$A = \frac{1}{Y_{21}}$, $B = \frac{Y_{22}}{Y_{21}}$

Comparing ⑥ & ②

$$C = -\frac{(Y_{11}Y_{22} - Y_{21}Y_{12})}{Y_{21}}$$

$D = \frac{Y_{11}}{Y_{21}}$

8) ABCD in terms of h parameters:

$V_1 = AV_2 - BI_2$ — ①

$I_1 = CV_2 - DI_2$ — ②

$V_1 = h_{11} I_1 + h_{12} V_2$ — ③

$I_2 = h_{21} I_1 + h_{22} V_2$ — ④

From ④

$I_1 = \frac{1}{h_{21}} I_2 - \frac{h_{22}}{h_{21}} V_2$.

$I_1 = \frac{-h_{22} V_2}{h_{21}} + \frac{1}{h_{21}} I_2$ — ⑤.

Sub ⑤ in ③

$V_1 = h_{11} \left(\frac{-h_{22}}{h_{21}} V_2 + \frac{1}{h_{21}} I_2 \right) + h_{12} V_2$

$= \frac{-h_{11} h_{22} V_2}{h_{21}} + \frac{h_{11}}{h_{21}} I_2 + h_{12} V_2$.

$V_1 = \frac{-(h_{11} h_{22} - h_{21} h_{12})}{h_{21}} V_2 + \frac{h_{11}}{h_{21}} I_2$. — ⑥

Comparing ① & ⑥.

$A = \frac{-(h_{11} h_{22} - h_{21} h_{12})}{h_{21}}$

$B = \frac{-h_{11}}{h_{21}}$

Comparing ② & ⑤.

$C = \frac{-h_{22}}{h_{21}}$, $D = \frac{-1}{h_{21}}$

7/8